1. Section 3.1 (Page 115-116): 6a (only for degrees one and two; ignore $x_3 = 0.75$), 8a, 19b

2. Section 3.3 (Page 133-136) 8, 11b (assume 11a is true), 17

3. This is a Matlab question. Before you start the problem, copy the following program to a file called interppoly.m in the directory you will be using when running matlab.

```
function [yout, cof] = interppoly(xin,yin,xout)
% This function first determines the vector of coefficients cof
% in the Newton form of the interpolating polynomial of degree
% less than or equal to n-1, i.e., the polynomial p(x) of degree
% less than or equal to n-1 of the form
% p(x) = cof(1) + cof(2)(x-xin(1)) + ... + cof(n)(x-xin(1))...(x-xin(n-1))
% satisfying p(xin(i)) = yin(i), i=1:n,
% where the inputs xin and yin are vectors of length n.
% The function then determines the values
% yout = p(xout), where xout is a vector of length m.
n = length(xin)
for k=1:n-1
yin(k+1:n) = (yin(k+1:n)-yin(k)) ./ (xin(k+1:n) - xin(k));
end
cof = yin;
yout = cof(n)*ones(1, length(xout));
% ones(l,m) returns an l-by-m matrix of 1s.
for k=n-1:-1:1
yout = (xout-xin(k)).*yout + cof(k);
end
```

   (a) Let $[a, b] = [-5, 5]$. For each $n = 4, 8, 16$, and 32, plot over the interval $[a, b]$ the error function $e(x) = f(x) - P_n(x)$, where $f(x) = \frac{1}{1+x^2}$ and $P_n(x)$ is the polynomial of degree $\leq n$ which interpolates f at the equally spaced interpolation points $x_i = a + \frac{b-a}{n} i$, for $i = 0, \ldots, n$. Record the approximate maximum of $|e(x)|$ (as seen from the graph) for each n and approximately where it occurs. To get you started, the case $n = 4$ can be done by typing (or cutting and pasting) the following commands into Matlab

```
% Problem 3a
a=-5; b=5;
n=4;
xin=linspace(a,b,n+1);
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,ytrue-yout)
```

(b) Repeat part (a), this time using the polynomial $Q_n(x)$ of degree $\leq n$ which interpolates f at the (Chebyshev) points

$$x_i = \frac{a+b}{2} + \frac{a-b}{2} \cos\left(\frac{(2i+1)\pi}{2n+2}\right), \qquad i = 0, \ldots, n$$

To get you started, the case n = 4 can be done by typing (or cutting and pasting) the following commands into Matlab.

```
% Problem 3b
close all % closes plotting windows
a=-5; b=5;
n=4;
x=linspace(1,2*n+1,n+1);
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xin = (a+b)/2 + (b-a)/2 * cos(x*pi/(2*n+2));
yin= 1./(1+xin.^2);
xout=linspace(a,b,100);
[yout, cof] = interppoly(xin,yin,xout);
ytrue= 1./(1+xout.^2);
plot(xout,ytrue-yout)
```

(c) Based on your plots, does the choice of interpolation points make a difference in the error in the approximation? Which choice is better in this case?